Granüler Malzemelerde Stabil Kemerlenme Oluşumuna Yönelik Bir Çalışma- Ayrık Elemanlar

A STUDY ON STABLE ARCH FORMATION IN GRANULAR MATERIALS – DISCRETE ELEMENTS METHOD

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ÖZET

Granüler malzemelerin makro ölçekteki davranışı büyük ölçüde parçacık seviyesindeki özelliklerinden etkilenir. Tek tek parçacıkların nasıl etkileşime girdiğindeki önemli faktörlerden biri, parçacıkların birbirine kenetlenmesine yol açan şekilleridir. Bu kenetlenme, kuvvetlerin parçacıklar arasında iletilme şeklini ve hareket etme biçimlerini etkileyerek malzemedeki akışı ve gerilim dağılımını etkiler. Ayrık eleman modellemesi (DEM) ve deneysel testler gibi sayısal yöntemler, parçacık şeklinin hem küçük ölçekli hem de büyük ölçekli davranışı nasıl etkilediğini anlamak için kullanılabilir. Granüler malzemelerdeki kemerlenme, siloların, hunilerin ve jeotekniklerin tasarımı dahil olmak üzere çeşitli mühendislik alanlarında karşılaşılan bir olgudur. Parçacıklar bir açıklıktan geçerken etkileşimleri ve kesme gerilimlerinin harekete geçmesi, parçacıkların akışını engelleyen kemer şeklinde bir oluşum oluşturur. Bu makalede, farklı kapak genişlikleri üzerinde kararlı kemerlerin oluşturulması DEM yoluyla sayısal olarak incelenmiştir. Bu çalışmada, Guo & Zhou (2013) ve Ahmadi & Hosseininia (2018) tarafından geliştirilen ayarlanabilir trapdoor genişliğine sahip fiziksel bir trapdoor test düzeneği EDEM'de modellenmiştir. 2D'de parçacıklar arası davranışı incelemek ve değişken sayısını azaltmak için özdeş cam boncuklar kullanılmıştır. Özdeş cam boncukların oluşturduğu gevşek ve yoğun düzenlemeler daha sonra, stabil kemerin çökmesine kadar 5 mm'lik adımlarla trapdoor genişliği değiştirilerek analiz edilmiştir.

Anahtar Kelimeler: *Granüler zeminler, Zeminde kemerlenme, Tuzak kapı deneyi, Ayrık elemanlar analizi, Micro zemin davranışı, Kohezyonsuz malzemeler*

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ABSTRACT

The behavior of granular materials at the macro scale is primarily influenced by their characteristics at the particle level. One key factor in how individual particles interact is their shape, which leads to particle interlocking. This interlocking impacts the way forces are transmitted between particles and how they move, affecting the flow and stress distribution in the material. Numerical methods, such as discrete element modeling (DEM) and experimental tests, could be used to understand how particle shape influences both small-scale and large-scale behavior. Arching in granular materials is a phenomenon encountered across various fields of engineering, including the design of silos, hoppers, and geotechnics. As particles move through an opening, their interactions and the mobilization of the shear stresses form an arch-shaped formation, which prevents the flow of particles. In this paper, the generation of stable arches over different trapdoor widths was investigated numerically through DEM. In this study, a physical trapdoor test setup developed by Guo & Zhou (2013) and Ahmadi & Hosseininia (2018) with adjustable trapdoor width was modeled in EDEM. In order to study the interparticle behavior in 2D and to reduce the number of variables, identical glass beads were used. The generated loose and dense arrangements of the identical glass beads were then analyzed by changing the trapdoor width in 5mm steps up to the collapse of the stable arch.

Keywords: *Granular materials, Soil arching, Trapdoor test, DEM analysis, Micro soil behavior, Cohesionless materials.*

1. INTRODUCTION

The Discrete Element Method (DEM) is commonly used to track particle interactions and gather detailed particle-level data. While different methods are used to represent particle shape in DEM, most studies use spherical particles because it simplifies contact detection and speeds up simulations. However, spherical particles do not interlock, so to better capture the effects of shape, researchers often add a rotational constraint at contact points, known as a rolling resistance model. The rolling resistance influences the material's flow, packing density, and stress patterns. With advances in computing power, the Discrete Element Method (DEM) has become more widely used (Cundall and Strack, 1979). In DEM, granular materials are treated as collections of individual particles, with their positions, speeds, and the forces they exert on each other being tracked at very short time intervals. Particle shape plays a crucial role in how individual particles interact, impacting their behavior at larger scales. Irregularly shaped particles tend to interlock, restraining their movement and altering the flow properties of the granular material. In DEM simulations, different methods are used to account for particle shape.

2. AIM OF THE CURRENT RESEARCH

Arching is a phenomenon in granular materials that appears in various areas of engineering, including the design of silos and hoppers, as well as in geotechnical engineering problems. As particles interact while flowing through an opening, an arch-like structure forms, causing the particles to become stationary. Arching, which results from the interaction between particles in granular materials, has been extensively studied in several geotechnical problems, such as in underground structures Chen et al., (2011), pile-reinforced soils

Abusharar et al., (2009), and Li et al., (2012), geosynthetic-reinforced layers Zhu et al., (2012), and earth pressure on retaining structures (Qiu & Grabe, 2012; Paik & Salgado, 2003). Due to the nature of the continuum mechanics, it is not possible to capture the discrete interparticle behavior occurring during the formation or collapse of the arches. Therefore, the study of the arching needs to be conducted using either physical model tests or DEM analyses.

The trapdoor test serves as an experimental method to assess the arching phenomenon in granular materials. Terzaghi (1936) was the first to carry out trapdoor experiments, characterizing the arching effect as the mechanism of load transfer through granular materials. Numerous other researchers have conducted similar tests for various objectives (Chevalier et al., 2012; Costa et al., 2009; Vardoulakis et al., 1981). This highlights the need for further experimental research on arch formation in different scenarios.

In this study, a trapdoor test setup designed by Guo & Zhou (2013), and Ahmadi & Hosseininia (2018) was modeled using EDEM to explore how stable arches develop in cohesionless granular materials more effectively. In the DEM simulation, the width of the trapdoor was adjusted in small increments, allowing for detailed observation of the changes in self-supporting stable arches prior to their collapse. This paper presents a sensitivity analysis of how spherical particles discharge when rolling resistance is applied. The findings provide insights into the macro behavior of these particles.

3. DEM ANALYSIS FOR THE TRAPDOOR MODEL

In DEM, the first step is to define the system's geometry, where particles are allowed to overlap when they come into contact. During each time step, particle positions are updated. If the distance between two particles is less than the sum of their radii, the particles overlap and are considered in contact. Different contact detection methods are used for more complex shapes (Podlozhnyuk et al., 2017). Once particles overlap, normal and tangential forces are calculated using a contact model. DEM applies Newton’s second law to determine the acceleration of each particle based on any unbalanced forces acting on it. This process links the forces at contact points to the resulting particle movements.

In real materials, rolling friction occurs when deformation occurs at the point of contact (which applies to elastic materials). However, while particles can overlap in DEM simulations, there is no actual deformation at the contact points. Therefore, rolling resistance needs to be introduced for particle-to-particle and particle-to-surface contacts. This is especially important for spherical particles with more rotational freedom because they do not interlock. Also, static friction could be defined simply as the force that opposes the movement between two surfaces in contact. The other material property in DEM is the restitution coefficient, defined as the energy loss at contact points.

The DEM model used in the analysis simulates the experimental study of Guo & Zhou (2013), and Ahmadi & Hosseininia (2018). The test setup consists of a wooden board measuring 550 × 400 × 12 mm, onto which a plexiglass panel, stationary parts, and a bottom compartment are mounted (Fig. 1). Two movable stationary parts are positioned at the bottom to create an opening for a trapdoor, which can be extended to 300 mm in width. The area defined by the main base, plexiglass sheet, side supports, and stationary parts forms a test box with dimensions of 300 × 400 mm. The thickness of the test box is adjustable to accommodate a single layer of material, simulating specific conditions as detailed by (Guo & Zhou, 2013).

Spherical glass beads were selected to represent cohesionless ideal granular materials. Images of these materials are shown in Fig. 2. The glass beads consist of uniform grains, each with a diameter of 12 mm. These beads are characterized as well-rounded, with sphericity (S) = 1, roundness (R) = 1, and regularity (ρ) = 1 (Tables 1 and 2).

For the spherical glass beads, two distinct particle arrangements were examined to model loose and dense states, as illustrated in Fig. 2. The variation in grain arrangement results in samples exhibiting different peak frictional angles due to the interlocking effect between the particles (Rowe, 1962). In the first configuration, representing a loose state, the grains were organized in a column-like arrangement, as depicted in Fig. 2c, with a void ratio of e = 0.27. The second configuration, representing a dense state shown in Fig. 2d, was arranged such that the center of each bead in even rows was positioned between two lower beads in the odd rows, achieving the minimum void ratio of e = 0.10.

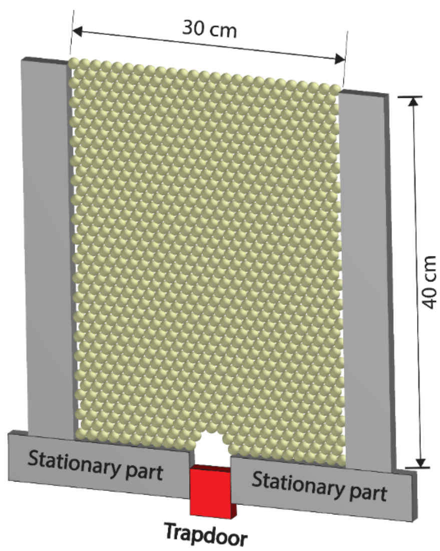

Figure 1. Trapdoor test setup in the EDEM.

Table 1. Characteristics of granular materials used in the tests.

Parameter			Beads (loose or dense state)
Diameter	d	[mm]	12
Solids density	ρ	[kg/m^3]	2500

Table 2. Material and model properties in DEM.

Parameter	Value
Density (particle) ρ [kg/m3]	2500
Coefficient of particle-particle friction μ^s_{pp}	0.35
Coefficient of particle-wall friction μ^s_{pw}	0.3
Coefficient of restitution (particle-particle), ε_p	0.8
Coefficient of restitution (particle-wall), ε_{pw}	0.5

Coefficient of restitution (particle-particle), ε_p	0.01
Coefficient of restitution (particle-wall), ε_{pw}	0.1
Poisson ratio (particles), ν_p	0.25
Poisson ratio (wall), ν_w	0.25
Shear modulus (particles), Gp[Pa]	10^8
Shear modulus (wall), G_w[Pa]	10^8
DEM model	Hertz- Mindlin (no-slip) with standard rolling friction

In this study, trapdoor tests were conducted on each type of granular material (glass beads in both loose and dense configurations). Each test consisted of a series of analyses, where the width of the trapdoor was gradually increased in 5 mm steps. The procedure for each run involved the following steps: (1) the stationary parts were adjusted symmetrically to ensure the trapdoor covered the intended opening. The test box was then filled with granular materials; (2) the trapdoor was slowly lowered (0.001 m/s) to allow the grains to discharge; (3) a statically stable arch structure might form above the trapdoor, stopping the flow of grains. The dimensions of this stable arch were recorded; (4) the next test run began with the trapdoor width increased by 5 mm, continuing until no stable arch formed on the stationary parts and the remaining grains flowed freely.

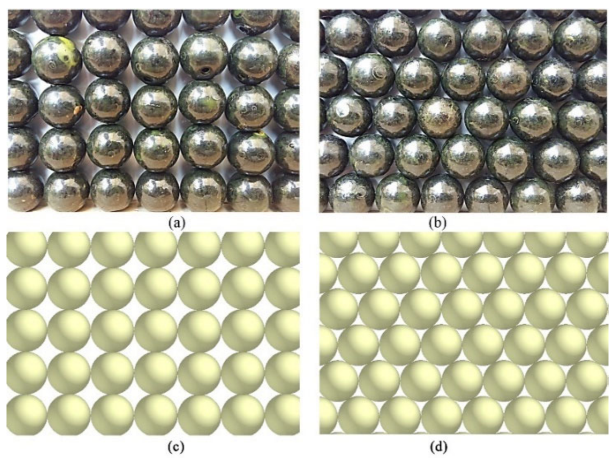

Figure 2. Particle arrangement: a) Ahmadi and Hosseninia's (2018) physical model – loose state, b) Ahmadi and Hosseninia's (2018) physical model – dense state, c) Current DEM analysis – loose state, d) Current DEM analysis – dense state.

4. OBSERVATIONS AND DISCUSSION

Figure 3 shows the stable arches formed over the stationary parts in the tested granular materials. Given that the grains are entirely cohesionless, the only factor contributing to arch formation is the interlocking between the particles. To more accurately examine the formation of statically stable arches over varying trapdoor widths, the geometry and dimensions of an arch are specified, as shown in Fig. 3. Each stable arch consists of a crown and two abutments, as depicted in Fig. 3. Also in Fig. 3, the trapdoor width is represented by the opening width (W), while the internal distance between the two abutments is referred to as the arch width (B). The vertical distance from the origin to the underside of the crown is defined as the arch height (H).

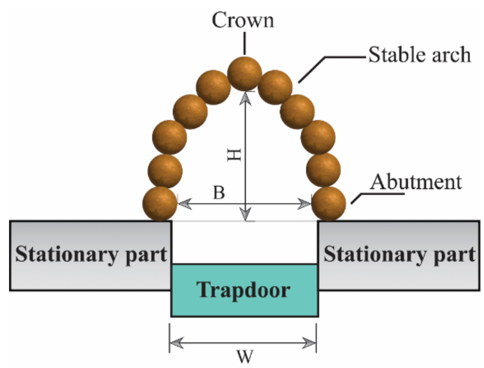

Figure 3. Definition of the geometry of the arches: arch width, height, and trapdoor width.

In all test series, the arch width (B) was recorded alongside the trapdoor width (W). The relationship between the arch width (B) and the trapdoor width (W) in the current DEM analysis compared to Ahmadi and Hosseninia's 2018 study is shown in Fig. 4a. It was observed that the arch width (B) does not always match the trapdoor width (W), with B ranging from 0.47 to 1.4 times W. This indicates that, for the materials tested in this study, the abutments of the arches are not always positioned at the trapdoor's edges but may form with some offset.

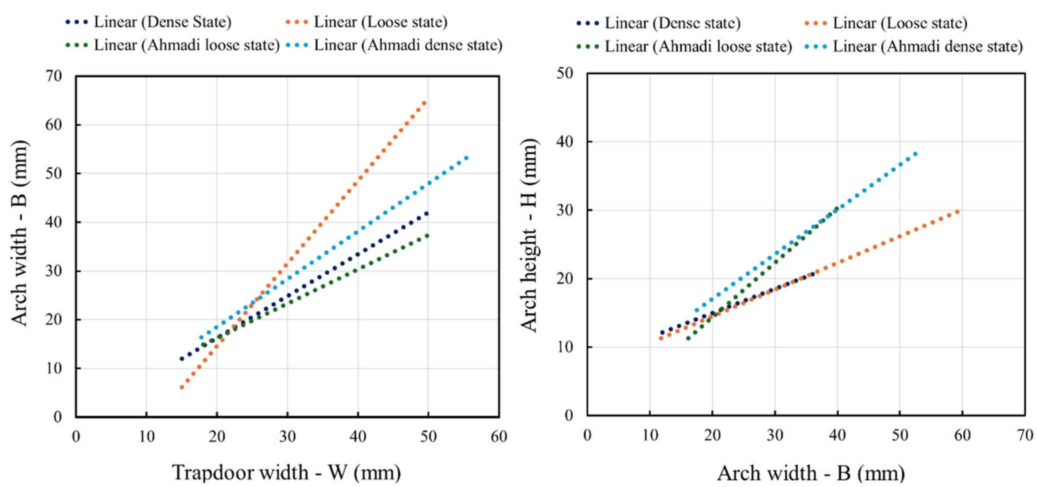

Figure 4. Variation of a. arch width; b. Arch height for different trapdoor widths

Fig. 4b illustrates how the arch height (H) varies with the arch width (B) in the current DEM analysis compared to Ahmadi and Hosseninia's 2018 study. The critical height, which represents the height of the last stable arch, is always lower than the height just before it. This indicates that as the trapdoor width increases, a drop in height signals the onset of arch collapse.

As can be seen, there is a distinct difference between the findings of the physical model and its DEM simulation. This difference mainly arises from adopting the different frictional properties in the DEM analysis compared to the physical model. In the DEM, the rolling and static friction between the front and back walls of the models with the beads are set to zero to decrease the boundary effect in the third dimension. Also, considering the bead's arrangement in the dense state of the physical model, it is evident that some particles lack complete senary contact; however, in the EDEM simulation, any bead has contact with all surrounding beads to avoid the contact-related shortcomings of the physical model. The study of stable arch formation and collapse in granular materials can be approached by examining the shear strength properties. From a micromechanical perspective, force chains between particles are crucial in creating arch structures within granular masses. Without cohesion, the stability of these arches relies entirely on the mobilized shear strength between particles. It is well established that in cohesionless granular materials, the shear strength (τ_f) follows the Coulomb friction law, represented by the formula: $\tau_f = \sigma_n \tan\phi_{mob}$, where σ_n is the normal stress, and ϕ_{mob} is the mobilized friction angle. The peak friction angle (ϕ_{peak}) defines the maximum shear strength at low deformation, while the critical state friction angle (ϕ_{cv}) determines the shear strength during large deformations. If the applied force causes the shear stress to surpass the material's shear strength (at the critical state), the arch structure will collapse.

Figure 5 presents the relationship between the arch height (H) and trapdoor width (W) in the current DEM analysis compared to Ahmadi and Hosseninia's 2018 study. Aside from the decrease in height linked to the critical arch, it is remarkable that, despite having identical physical properties, the dense beads have a growth rate smaller than that of the loose beads. This observation can be explained by the fact that, in statically stable arches, maximum load transfer occurs among particles, leading the mobilized friction angle to align with the peak friction angle. Because the beads are in the most compacted state in the dense state, the internal friction angle is at its highest value. However, for a stable arch to be formed in the loose state, the beads should move and roll over each other to get into a denser state and cause jamming over the trapdoor void.

Figure 6 illustrates the DEM analysis results for the test beads' loose and dense state arrangement. The arch width is not necessarily equal to the trapdoor. Also, there is no possibility for the particles to be densified at the dense state, so by lowering the trapdoor, the particles immediately over the trapdoor will discharge, but the whole system would get into equilibrium and stay intact. However, in the loose state, after lowering the trapdoor, a higher number of beads would leave the test box, and the whole system would be in a turbulent phase before getting into a new equilibrium by forming a stable arch (Fig 6. a and b). Figures 6 c and d show the intensity and direction of the stress tensor in the vertical plane. The direction of the stress tensor is shown in red and black for the positive and negative sides of the x-axis of the coordinate system. Also, the fading effect in the colors represents the intensity of the stress. In both cases, the concentration of the stresses over the stationary parts and the arched region is recognizable. Figures 6 e and f depict the contact normal force direction that could be interpreted as proof of principal stress rotation, which took the shape of an arch over the lowered trapdoor. In the end, the direction of the tangential force is shown in Figures 6 g and h. For the dense state due to the regular hexagonal arrangement of the beads in the dense state, the tangential force direction makes a honeycomb-like force chain. However, for the loose state, due to the arbitrary and chaotic movement of the beads during the discharge and before the formation of the stable arch, no meaningful force chain shape could be noticed.

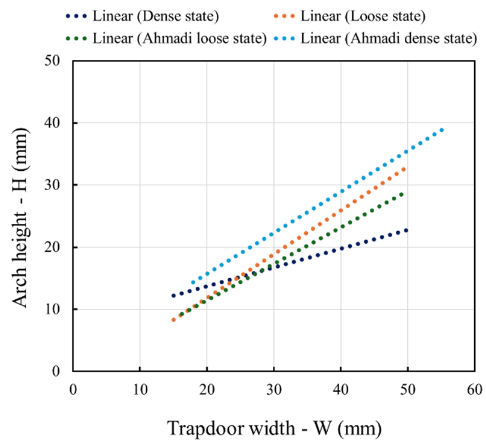

Figure 5. Variation of arch height versus the trapdoor width.

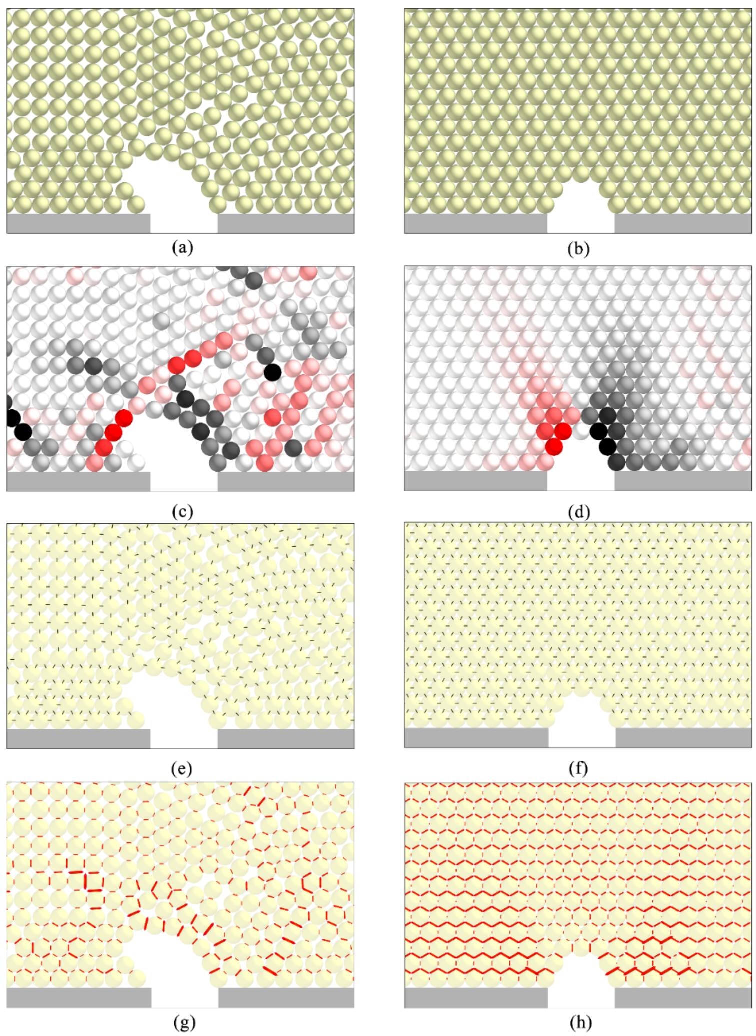

Figure 6. DEM analysis results for loose (left column) and dense states (right column): a) and b) Arch geometry, c, and d) Stress tensor in the vertical plane, e, and f) Contact normal force direction, g, and h) Tangential force direction.

5. CONCLUSION

In this paper, the generation of stable arches over different trapdoor widths was investigated numerically through DEM. A trapdoor test setup developed by Guo and Zhou and Ahmadi and Hosseininia with adjustable trapdoor width was modeled in EDEM. A loose and dense arrangement of identical glass beads was generated, and the trapdoor width was increased in 5mm steps up to the collapse of the stable arch.

- Statically stable arches can form above a trapdoor when the ratio of trapdoor width to bead diameter (W/d) falls within the range of 1.25 to 4.2.
- For the materials tested, the arch width does not necessarily match the trapdoor width; a stable arch may form with either a greater or smaller width than the trapdoor.
- The peak friction angle plays a critical role in determining both the stability and the dimensions of these statically stable arches.
- The height of the final arch before collapse is not the highest; the arch height decreases prior to failure.
- A direct correlation exists between the width and height of stable arches, and this relationship depends on the material's density.
- The onset of arch instability, or collapse, is tied to the shear strength of the granular material, particularly when the material undergoes significant deformation, where the critical state friction angle defines the friction angle.

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